# An Equivalent Cardiac Generator which Preserves Topography

Dear Sir:

In reference to Plonsey's (1) excellent discussion of the "inverse problem" in electrocardiography, I would like to point out a representation of the cardiac generator which shares some characteristics with the multipole expansion, available in principle from the body surface potentials, and with the desired but elusive actual generator distribution.

Helmholtz (2) proved that a generator distribution can be replaced, as far as the field exterior to it is concerned, by a closed nonuniform double layer on any surface bounding the distribution. As I have already indicated (3), this representation for any *particular* surface surrounding the generator distribution is unique except for a double layer of constant moment. This uniqueness may be easily seen by the following arguments. Were two representations to exist, according to the superposition theorem we would expect that the double layer, whose strength at any point is given by the difference between those of the representations, would produce no external field or potential. The continuity of the field normal to the surface containing the double layer then implies that this field component also vanishes over the inside of the surface. The uniqueness of the solutions to the Neumann problem, for the volume enclosed by this surface, ensures us that the potential everywhere within the surface is constant, and the uniformity of the double layer, comprising the difference between the representations, follows immediately.

Such a unique double layer representation exists if the volume conductor is not homogeneous in resistivity, and, in fact, even in the special case where this inhomogeneity distinguishes the "heart" from the rest of the body tissue. The double layer equivalent generator is thus the three dimensional analogue of that guaranteed to circuit theory by Thevenin's theorem.

Since electrogenesis in the heart is certainly limited to the myocardium, one can arbitrarily choose a closed surface surrounding this muscle and fitting it as tightly as desired for the double layer. The instantaneous multipole expansion around an origin located within the surface can, in principle, be determined from surface potentials as described by Geselowitz (4). The potential, defined by this only outside a sphere surrounding the heart, can be determined at the double layer surface by analytic continuation. Then, the double layer generator corresponding to this can be obtained by solving the integral equation:

$$\Phi(\mathbf{r}) = \frac{\mu(\mathbf{r})}{2} + \frac{1}{4\pi}\int_S \mu(\boldsymbol{\varrho})\left[\frac{\mathbf{n}(\boldsymbol{\varrho})\cdot(\mathbf{r}-\boldsymbol{\varrho})}{|\mathbf{r}-\rho|^3} + \frac{1}{\mathbf{r}}\right] dS$$

where $\mu$ is the desired double layer movement, $\Phi$, the potential defined by the multipole expansion, $\mathbf{n}$ the surface normal, and $\mathbf{r}$ and $\boldsymbol{\varrho}$ position vectors to surface points. The kernel of the integral is chosen to eliminate the arbitrary additive constant in double layer strength, as described by Mikhlin (5).

Practically, one can find the first few terms of the multipole expansion under the restrictions indicated by Plonsey (1) and assume that they themselves provide a good picture of the potential at the double layer surface. Then this potential can be used to find the equivalent double layer distribution at the model surface. Interestingly enough, the solution of the same integral equation with the surface of integration altered to conform to the torso surface, provides a means of translating potentials measured on this free surface into those that would be present if the body were an infinite volume conductor.

I have written and begun to test a computer program, based on the above considerations, which will permit the transformation of potentials measured at, say, 17 points on the torso, into the equivalent double layer strengths at 16 points on a convex surface fitting the heart. This transformation could be done on line, as the surface potentials are being recorded, by an appropriate real time computer.

Although the equivalent double layer generator computed in the above fashion suffers from the same sensitivity to the location of the origin that the truncated multipole expansion does (Plonsey, reference 1) the former has the great advantage over the latter of yielding some localizing information about the real instantaneous cardiac generators. The closer to the model surface these actual generators are, the better should the double layer distribution indicate their extent. On the other hand, the computation of this double layer model is not dependent on the arbitrary choice of dipole positions within the heart, as in the multiple dipole model discussed by Plonsey. I would argue that the a priori disposition of the model surface for the double layer is based on fewer assumptions than this choice of dipole positions. In principle, the double layer model would give the cardiologist exactly the same information, and in the identical form, as would be available from potential measurements at the outer surface of an isolated heart.

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## REFERENCES

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Leonard Zablow
Department of Neurology
College of Physicians and Surgeons
Columbia University
New York, New York